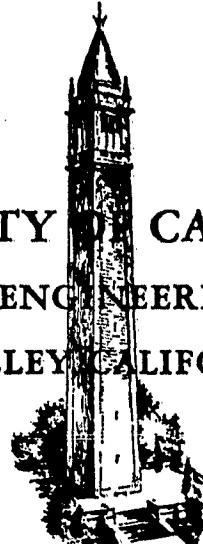


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REPORT NO. HE-150-115

TECHNICAL REPORT

UNIVERSITY OF CALIFORNIA  
INSTITUTE OF ENGINEERING RESEARCH  
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HEAT TRANSFER TO A CYLINDER FOR THE FREE  
MOLECULE FLOW OF A NON-UNIFORM GAS

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By

S. BELL

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FLUID FLOW AND  
HEAT TRANSFER  
AT LOW PRESSURES  
AND TEMPERATURES

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HEAT TRANSFER TO A CYLINDER FOR THE FREE  
MOLECULE FLOW OF A NON-UNIFORM GAS

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By

S. BELL

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Approved by:

S. A. SchAAF

## NOMENCLATURE

$dA$	= element of surface area
$C_p$	= specific heat at constant pressure
$d$	= cylinder diameter
$dE_i, dE_r, dE_w$	= molecular energy fluxes incident, reflected and diffusely reflected from $dA$ per unit time, Eq. 2.2
$f$	= molecular velocity distribution function, Eq. 2.5
$f^o$	= Maxwellian distribution function, Eq. 2.6
$f_0, f_1, \dots, f_j$	= factors of the velocity distribution function, Eq. 2.12
$H_o, H_{p_{xx}},$ $H_{p_{yy}}, H_{q_x}$	= functions of $s$ defined by Eq. 3.5
$h$	= heat transfer coefficient per unit area, Eq. 3.11
$I_0, I_1$	= modified Bessel functions of zero and first order
$j$	= number of internal degrees of freedom
$K$	= Boltzmann's constant
$L$	= length of cylinder
$M$	= Mach number ( $U/V_a$ )
$m$	= molecular mass
$dN_i, dN_w$	= number of molecules incident on and diffusely reflected from $dA$ per unit time, Eq. 2.9
$Nu$	= Nusselt number ( $hd/\kappa$ )

$p$  = pressure  
 $\Pr$  = Prandtl number ( $c_p \mu / \kappa$ )  
 $\rho_{xx}, \rho_{yy}, \rho_{zz}$  = normal stress deviation terms, Eq. 2.5  
 $dQ$  = heat removed from  $dA$  by sinks inside the cylinder (per unit time)  
 $Q$  = total heat removed from the cylinder (per unit time)  
 $g_x, g_y, g_z$  = heat flux terms, Eq. 2.5  
 $R$  = gas constant  
 $Re$  = Reynolds number ( $\rho U d / \mu$ )  
 $dR_i, dR_r$  = radiant energy fluxes incident on, and reflected from,  $dA$  per unit time, Eq. 2.1  
 $r$  = recovery factor, Eq. 3.6  
 $r_o, r_{\rho_{xx}}, r_{\rho_{yy}}, r_{g_x}$  = partial recovery factors, Eq. 3.10  
 $S$  = molecular speed ratio, Eq. 3.3  
 $S_T$  = Stanton number ( $Nu / Re \Pr$ )  
 $S_{T_o}, S_{T_{\rho_{xx}}}, S_{T_{\rho_{yy}}}, S_{T_{g_x}}$  = partial Stanton numbers, Eq. 3.15  
 $T$  = gas temperature  
 $T_{aw}$  = equilibrium temperature of the cylinder wall in the absence of heat transfer, Eqs. 3.4 and 3.6  
 $T_o$  = stagnation temperature  
 $T_w$  = cylinder wall temperature  
 $U, V, W$  = gas velocity components

$u, v, w$  = molecular velocity components in  $x, y, z$ , direction, Fig. 1

$u', v', w'$  = molecular velocity components in  $x', y', z'$ , direction, Fig. 1

$V_1, V_2, \dots, V_j$  = characteristic velocities associated with various degrees of freedom, Eq. 2.11

$V_a$  = acoustic speed ( $\sqrt{\gamma RT}$ )

$x, y, z$  = coordinates referred to flow direction, Fig. 1

$x', y', z'$  = coordinates referred to surface element, Fig. 1

$\alpha$  = thermal accommodation coefficient, Eq. 2.2

$\gamma$  = ratio of specific heats

$\theta$  = angle between surface normal and flow direction, Fig. 1

$K$  = thermal conductivity

$\mu$  = viscosity

$\rho$  = gas density

$\tau_{xy}, \tau_{xz}, \tau_{yz}$  = shear stress terms, Eq. 2.5

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1.0 INTRODUCTION

The heat transfer to a body in free molecule flow has been determined theoretically by Tsien (Ref. 1), Stalder and Jukoff (Ref. 2) and others (Refs. 3 and 4). Experiments by Stalder, Creager and Goodwin (Refs. 3 and 5) have provided some confirmation of the theoretical developments.

These investigations have been confined to gas flows in Maxwellian equilibrium. The present report is a continuation of Reference 6 in which the aerodynamic forces on a cylinder were determined for the case of a non-uniform gas (i.e., one in which heat flux or viscous stresses are present).

It is assumed that the right circular cylinder, which is oriented perpendicular to the gas flow, has so large a thermal conductivity that it has a uniform wall temperature. Possible end effects are neglected.

The results depend on a knowledge of the molecular velocity distribution and the distribution assumed here is the one proposed by Grad (Ref. 7). The results are complete for the monatomic case but only approximate for the polyatomic case.

For the sake of brevity, much of the general discussion given in Reference 6 has not been repeated here.

2.0 METHOD OF ANALYSIS

2.1 Monatomic Gas

The general method of analysis is the same as in Reference 6 and the energy balance for a differential area is written

$$dE_i + dR_i = dE_r + dR_r + dQ \quad (2.1)$$

Assuming no radiation and using the thermal accommodation coefficient

$$\alpha = \frac{dE_i - dE_r}{dE_i - dE_w} \quad (2.2)$$

one may rewrite the energy balance for convection

$$\frac{1}{\alpha} dQ = dE_i - dE_w \quad (2.3)$$

The incident energy flux per unit area per unit time may be expressed

$$\frac{dE_i}{dA} = \frac{m}{2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{0}^{\infty} (u'^2 + v'^2 + w'^2) u' f du' dv' dw' \quad (2.4)$$

where  $f$  is the distribution function used in Reference 6,

$$f = f^0 \left[ 1 + \frac{1}{2pRT} \left\{ p_{xx}(u-U)^2 + p_{yy}v^2 + p_{zz}w^2 + 2\tau_{xy}(u-U)v - 2\tau_{xz}(u-U)w - 2\tau_{yz}vw - 2 \left[ 1 - \frac{(u-U)^2 + v^2 + w^2}{5RT} \right] \left[ q_x(u-U) + q_yv + q_zw \right] \right\} \right] \quad (2.5)$$

and

$$f^0 = \frac{p}{m(2\pi RT)^{3/2}} e^{-\frac{(u-U)^2 + v^2 + w^2}{2RT}} \quad (2.6)$$

Except for the molecular velocity components, all the variables appearing in Eqs. 2.5 and 2.6, are treated as constants throughout the region occupied by the cylinder. The relations between velocity components are given by

$$\left. \begin{aligned} u &= -u' \cos \theta + v' \sin \theta \\ v &= -u' \sin \theta - v' \cos \theta \\ w &= w' \end{aligned} \right\} \quad (2.7)$$

The energy flux per unit area per unit time due to diffusely reflected molecules is

$$\frac{dE_w}{dA} = 2 k T_w \frac{dN_w}{dA} \quad (2.8)$$

and since  $dE_w/dA$  is to be computed on the assumption that all of the molecules are reflected diffusely

$$\frac{dN_w}{dA} = \frac{dN_i}{dA} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} u' f du' dv' dw' \quad (2.9)$$

Combining Eqs. 2.3, 2.4, 2.8 and 2.9 and integrating over the cylinder

$$\frac{Q}{\alpha} = \frac{m dL}{4} \int_0^{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (u'^2 + v'^2 + w'^2 - 4 R T_w) u' f du' dv' dw' d\theta \quad (2.10)$$

## 2.2 Polyatomic Gas

In a polyatomic gas there are internal energies of rotation or of vibration or of both. To calculate the incident energy flux in this case, one should use a distribution function which includes the distribution of rotational and vibrational energies (e.g., see Ref. 8). If the gas molecules have  $j+3$  degrees of freedom of motion (thus,  $j = 0$  for a monatomic gas) the number of incident molecules may be written in the form

$$\frac{dN_i}{dA} = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} u' f(u', v', w', v_1, \dots, v_j) \times du' dv' dw' dv_1 \dots dv_j \quad (2.11)$$

where, for the sake of simplicity in illustrating a point, the classical mechanical approach has been adopted and a characteristic velocity has been associated with each degree of freedom of motion. Actually, an accurate calculation would require a quantum-mechanical formulation.

Assuming that the total energy can be expressed as a sum of energies of each degree of freedom, the distribution function for Maxwellian equilibrium may be expressed in factored form,

$$f(u', v', w', V_1, \dots V_j) = f_0(u', v', w') f_1(V_1) \dots f_j(V_j) \quad (2.12)$$

and the number per unit area of diffusely reflected molecules takes the form

$$\frac{dN_w}{dA} = \int_{-\infty}^{\infty} f_j(V_j) dV_j \times \dots \int_{-\infty}^{\infty} f_1(V_1) dV_1 \times \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} u' f_0(u', v', w') du' dv' dw' \quad (2.13)$$

This separation of variables also occurs in the integration for the energy flux per unit area due to diffusely reflected molecules.

Making the further assumption that the energy associated with each degreee of freedom has a quadratic dependence on the velocity involved, a straight-forward calculation yields, for the diffusely reflected energy flux (Ref. 9, p. 93)

$$\frac{dE_w}{dA} = \left[ 2kT_w + \frac{jkT_w}{2} \right] \frac{dN_w}{dA} \quad (2.14)$$

For the non-Maxwellian case, one cannot factor the distribution function as in Eq. 2.12 and the corresponding relations for the incident energy flux presumably would become very complicated. As a first approximation, however, it seems reasonable to assume a similar separation of translational and internal energies so that

$$\frac{dE_i}{dA} = \frac{m}{2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (u'^2 + v'^2 + w'^2) u' f du' dv' dw' + \frac{jKT}{2} \frac{dN_i}{dA} \quad (2.15)$$

Combining Eqs. 2.3, 2.9, 2.14, and 2.15, and integrating over the cylinder

$$\frac{Q}{\alpha} = \frac{mdL}{4} \int_{0}^{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (u'^2 + v'^2 + w'^2 + jRT - (4+j)RT_w) u' f du' dv' dw' d\theta \quad (2.16)$$

### 3.0 RESULTS

Performing the integrations indicated in Eq. 2.16 one has

$$\begin{aligned} Q = & \frac{\sqrt{\pi} d L \rho U R T \alpha}{4 s e^{s^2/2}} \left[ \left\{ [2s^4 + (7+j)s^2 \right. \right. \\ & \left. \left. + (4+j)] I_0(s^2/2) + [2s^4 + (5+j)s^2] I_1(s^2/2) \right\} \right. \\ & \left. + \frac{\rho_{xx}}{4\rho} \left\{ [8s^2 + (6+j)] I_0(s^2/2) + [8s^2 - (4+j)] I_1(s^2/2) \right\} \right. \\ & \left. + \frac{\rho_{yy}}{4\rho} \left\{ [2s^2 + (6+j)] I_0(s^2/2) + [2s^2 + (4+j)] I_1(s^2/2) \right\} \right. \\ & \left. + \frac{g \times s^2}{5\rho U} \left\{ [10-j] I_0(s^2/2) + [12+j] I_1(s^2/2) \right\} \right] \\ & - \frac{\sqrt{\pi} d L \rho U R T_w \alpha (4+j)}{4 s e^{s^2/2}} \left[ \left\{ (1+s^2) I_0(s^2/2) \right. \right. \\ & \left. \left. + s^2 I_1(s^2/2) \right\} + \frac{\rho_{xx}}{4\rho} \left\{ I_0(s^2/2) - I_1(s^2/2) \right\} \right. \\ & \left. + \frac{\rho_{yy}}{4\rho} \left\{ I_0(s^2/2) + I_1(s^2/2) \right\} \right. \\ & \left. - \frac{g \times s^2}{5\rho U} \left\{ I_0(s^2/2) - I_1(s^2/2) \right\} \right] \quad (3.1) \end{aligned}$$

where  $p_{xx}$  has been eliminated by the relation

$$p_{xx} + p_{yy} + p_{zz} = 0 \quad (3.2)$$

$I_0$  and  $I_1$  are the modified Bessel functions and  $S$  is the molecular speed ratio

$$S = \frac{U}{\sqrt{2RT}} = \sqrt{\frac{1}{2}} M \quad (3.3)$$

Eq. 3.1 is valid for a monatomic gas ( $j = 0$ ) and, under the assumptions of Eqs. 2.14 and 2.15, would also be valid for a polyatomic gas. The results of Eq. 3.1 agree with those of Refs. 3 and 4 for the uniform case. It will be observed that there are no terms corresponding to a shear stress  $T_{xy}$ , nor to a heat flux,  $q_y$ , in the transverse direction. This was not the case for the aerodynamic force calculations of Ref. 6. Since these two terms are the principal non-uniformities in a boundary layer flow, it follows that a heat transfer type of free molecule probe is particularly advantageous for boundary layer surveys.

If, by  $T_{aw}$ , one denotes the equilibrium temperature assumed by the cylinder in the absence of heat transfer (adiabatic wall temperature), then Eq. 3.1 may be solved for  $T_{aw}/T$  as a function of  $p_{xx}/p$ ,  $p_{yy}/p$  and  $q_x/pU$ . In the resulting expression for  $T_{aw}/T$  the denominator is expanded in powers of  $p_{xx}/p$ ,  $p_{yy}/p$  and  $q_x/pU$ . By neglecting squares and higher powers, one obtains, for a monatomic gas.

$$\frac{T_{aw}}{T} = H_0 + \frac{p_{xx}}{p} H_{p_{xx}} + \frac{p_{yy}}{p} H_{p_{yy}} + \frac{q_x}{pU} H_{q_x} \quad (3.4)$$

where:

$$H_0 = H_0(s) = \frac{F_0(s)}{G_0(s)}$$

$$H_{p_{xx}} = H_{p_{xx}}(s) = \frac{F_{p_{xx}}(s)}{G_0(s)} - \frac{F_0(s)G_{p_{xx}}(s)}{[G_0(s)]^2}$$

$$H_{p_{yy}} = H_{p_{yy}}(s) = \frac{F_{p_{yy}}(s)}{G_0(s)} - \frac{F_0(s)G_{p_{yy}}(s)}{[G_0(s)]^2}$$

$$H_{q_x} = H_{q_x}(s) = \frac{F_{q_x}(s)}{G_0(s)} - \frac{F_0(s)G_{q_x}(s)}{[G_0(s)]^2}$$

(3.5)

$$F_0(s) = (2s^4 + 7s^2 + 4)I_0(s^2/2) + (2s^4 + 5s^2)I_1(s^2/2)$$

$$F_{p_{xx}}(s) = (2s^2 + 3/2)I_0(s^2/2) + (2s^2 - 1)I_1(s^2/2)$$

$$F_{p_{yy}}(s) = (s^2/2 + 3/2)I_0(s^2/2) + (s^2/2 + 1)I_1(s^2/2)$$

$$F_{q_x}(s) = 2s^2 I_0(s^2/2) + (12s^2/5)I_1(s^2/2)$$

$$G_0(s) = (4s^2 + 4)I_0(s^2/2) + 4s^2 I_1(s^2/2)$$

$$G_{p_{xx}}(s) = I_0(s^2/2) - I_1(s^2/2)$$

$$G_{p_{yy}}(s) = I_0(s^2/2) + I_1(s^2/2)$$

$$G_{q_x}(s) = -(4s^2/5)I_0(s^2/2) - (4s^2/5)I_1(s^2/2)$$

The functions  $H_0$ ,  $H_{p_{xx}}$ ,  $H_{p_{yy}}$  and  $H_{q_x}$  are presented graphically in Fig. 2.

The recovery factor is defined by

$$R = \frac{T_{aw} - T}{T_0 - T} \quad (3.6)$$

where  $T_0$  is the stagnation temperature. Using the relation

$$\frac{T_0}{T} = 1 + \frac{\gamma-1}{\gamma} s^2 \quad (3.7)$$

one obtains

$$r = \frac{\gamma}{\gamma-1} \cdot \frac{1}{s^2} \left[ \frac{T_{aw}}{T} - 1 \right] \quad (3.8)$$

or using Eq. 3.4

$$r = \frac{\gamma}{\gamma-1} \left[ \frac{H_0 - 1}{s^2} + \frac{p_{xx}}{p} \cdot \frac{H_{p_{xx}}}{s^2} + \frac{p_{yy}}{p} \cdot \frac{H_{p_{yy}}}{s^2} + \frac{g_x}{pU} \cdot \frac{H_{g_x}}{s^2} \right] \quad (3.9)$$

Eq. 3.9 may be written

$$r = r_0 + \frac{p_{xx}}{p} r_{p_{xx}} + \frac{p_{yy}}{p} r_{p_{yy}} + \frac{g_x}{pU} r_{g_x} \quad (3.10)$$

These "partial recovery factors" are presented graphically, for a monoatomic gas, in Fig. 3.

One defines the heat transfer coefficient per unit area as

$$h = \frac{Q}{\pi d L (T_{aw} - T_w)} \quad (3.11)$$

and the Stanton number is

$$ST = \frac{Nu}{Pr Re} = \frac{\gamma-1}{\gamma} \cdot \frac{h}{\rho U R} \quad (3.12)$$

Combining Eqs. 3.1, 3.11 and 3.12

$$\begin{aligned}
 \frac{1}{\alpha} S_T = & \frac{\gamma+1}{\gamma} \cdot \frac{1}{4\sqrt{\pi} s e^{s^2/2}} \left[ (1+s^2) I_0(s^2/2) \right. \\
 & + s^2 I_1(s^2/2) + \frac{p_{xx}}{4\rho} \left\{ I_0(s^2/2) \right. \\
 & \left. - I_1(s^2/2) \right\} + \frac{p_{yy}}{4\rho} \left\{ I_0(s^2/2) + I_1(s^2/2) \right\} \\
 & \left. - \frac{g_x s^2}{5\rho U} \left\{ I_0(s^2/2) - I_1(s^2/2) \right\} \right] \quad (3.13)
 \end{aligned}$$

where use has been made of

$$\gamma = 1 + \frac{2}{j+3} \quad (3.14)$$

Eq. 3.13 may be written

$$\frac{1}{\alpha} S_T = \frac{1}{\alpha} S_{T_0} + \frac{p_{xx}}{\rho} \frac{1}{\alpha} S_{T_{p_{xx}}} + \frac{p_{yy}}{\rho} \frac{1}{\alpha} S_{T_{p_{yy}}} - \frac{g_x}{\rho U} \frac{1}{\alpha} S_{T_{g_x}} \quad (3.15)$$

These "partial Stanton numbers" are presented graphically, for a monatomic gas in Fig. 4.

Inspection of Fig. 4, reveals that the partial Stanton numbers corresponding to the non-uniform terms are all small compared to the Stanton number,  $S_{T_0}$ , for the uniform case. Unless a gas flow is very non-uniform, i.e., if the flow itself is in the continuum or slip-flow region, the quantities  $p_{xx}$  and  $p_{yy}$  are small compared to  $\rho$ . It follows that the energy balance will be effected only slightly by the contributions arising from the viscous stress terms.

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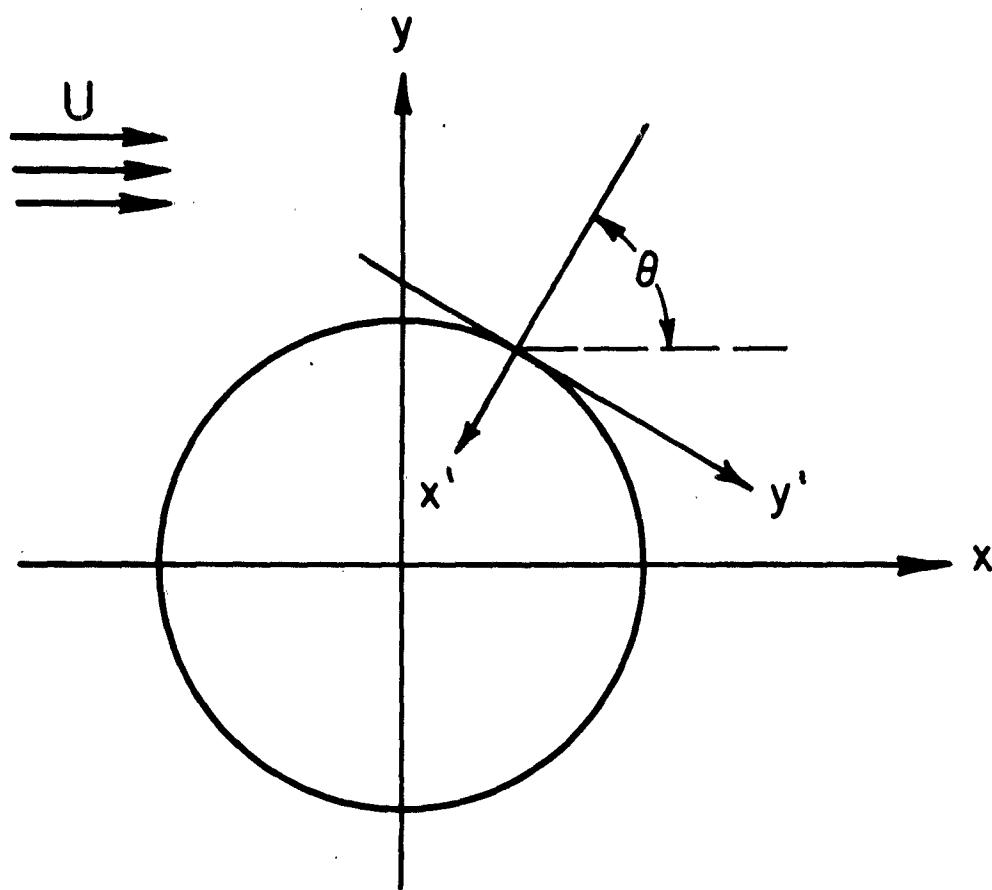


FIG. I COORDINATE SYSTEMS

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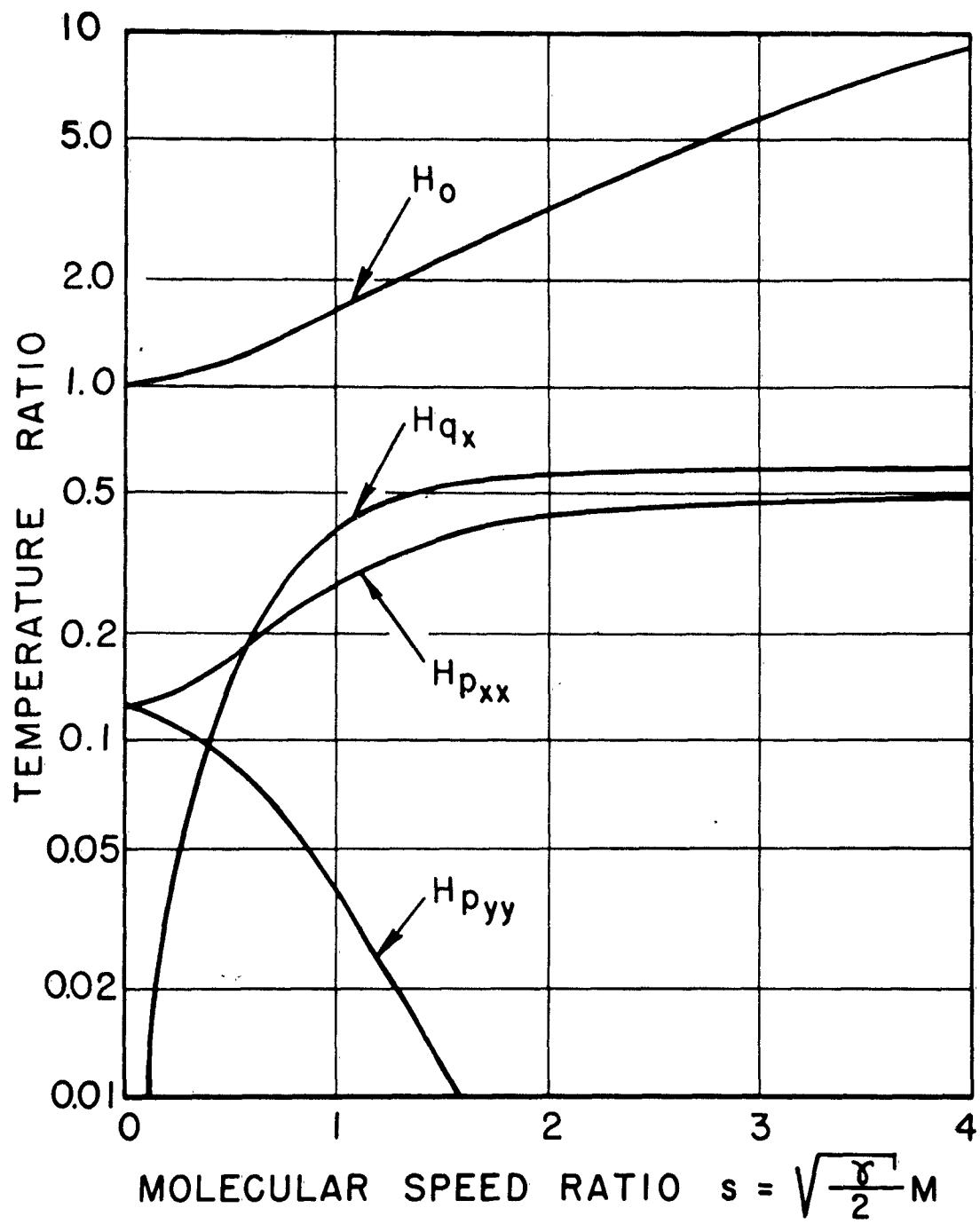


FIG. 2 PARTIAL TEMPERATURE RATIOS FOR  
NON-UNIFORM FREE MOLECULE FLOW  
PAST A CYLINDER (MONATOMIC GAS)

$$\frac{T_{aw}}{T} = H_0 + \frac{p_{xx}}{p} H_{p_{xx}} + \frac{p_{yy}}{p} H_{p_{yy}} + \frac{q_x}{pU} + H_{q_x}$$

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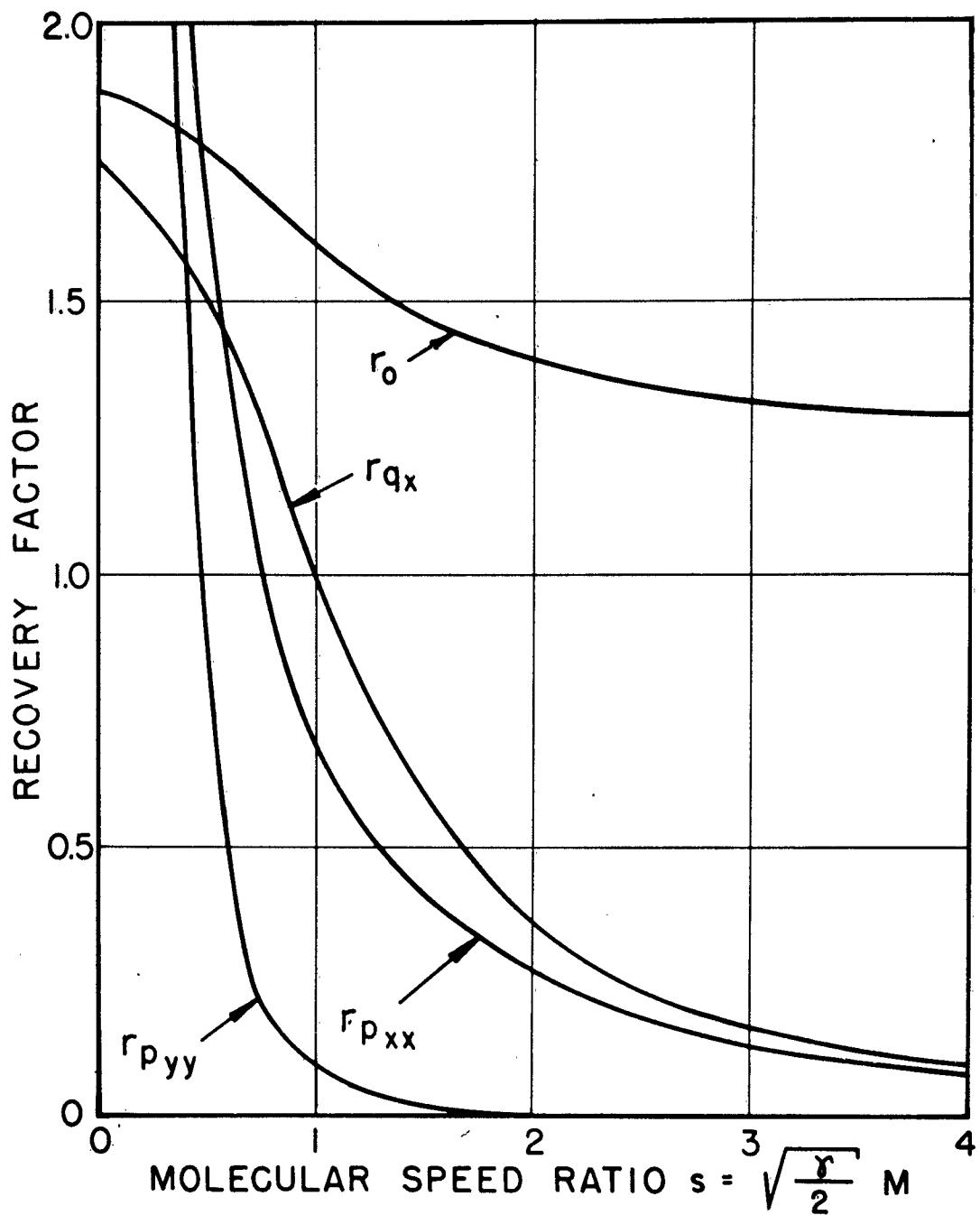


FIG. 3 PARTIAL RECOVERY FACTORS FOR  
NON-UNIFORM FREE MOLECULE FLOW  
PAST A CYLINDER (MONATOMIC GAS)

$$r = \frac{T_{\infty} - T}{T_0 - T} = r_0 + \frac{p_{xx}}{p} r_{p_{xx}} + \frac{p_{yy}}{p} r_{p_{yy}} + \frac{q_x}{pU} r_{q_x}$$

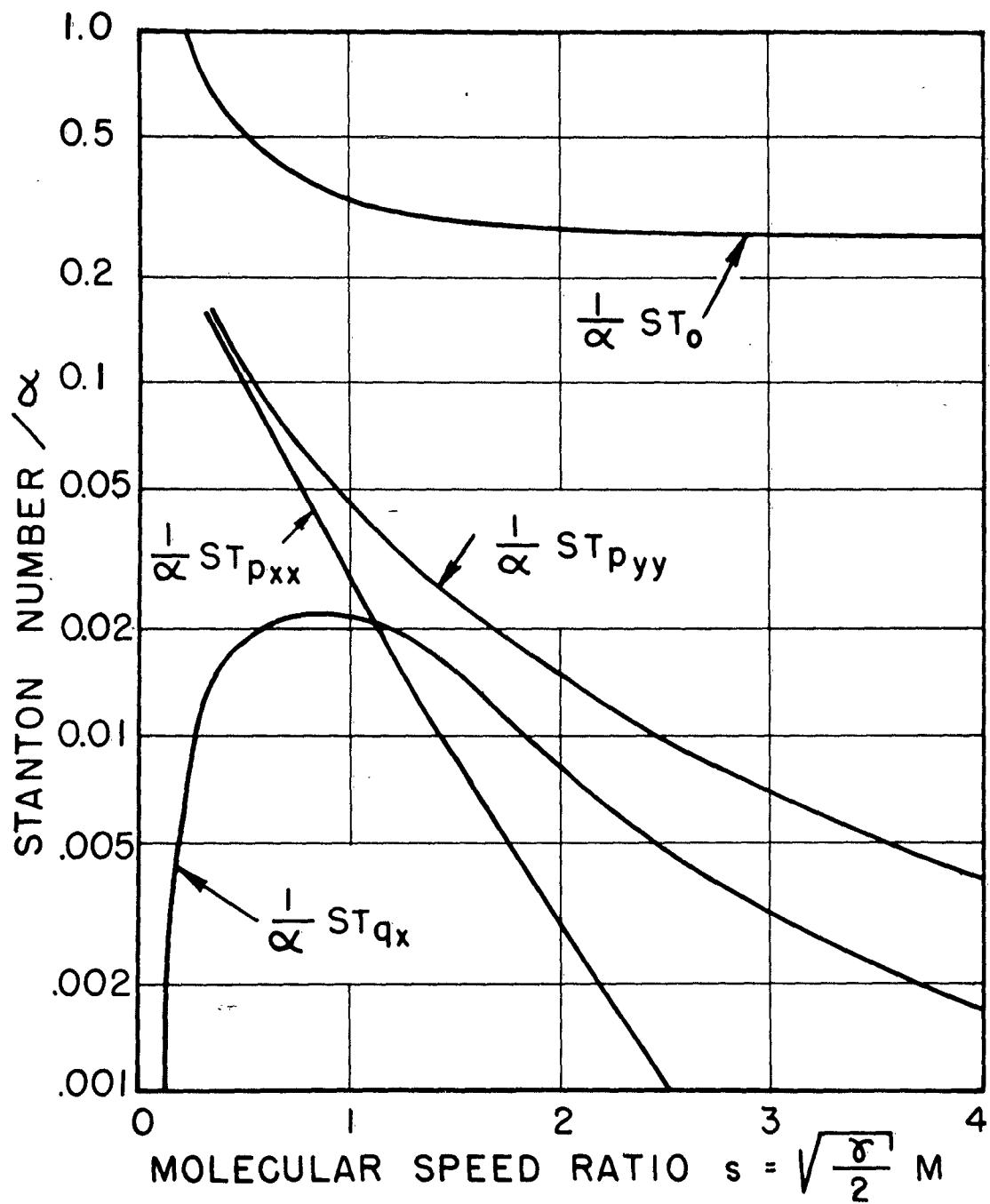


FIG. 4 PARTIAL STANTON NUMBERS FOR  
NON-UNIFORM FREE MOLECULE FLOW  
PAST A CYLINDER (MONATOMIC GAS)

$$\frac{1}{\alpha} ST = \frac{1}{\alpha} ST_0 + \frac{p_{xx}}{p} \frac{1}{\alpha} ST_{pxx} + \frac{p_{yy}}{p} \frac{1}{\alpha} ST_{pyy} - \frac{q_x}{pU} \frac{1}{\alpha} ST_{qx}$$

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